Characteristic Analysis of Directly Coupled HTS dc-SQUID Magnetometer Considering Josephson Junction as Equivalent Circuit

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Abstract — A cover of a SQUID ring with a superconducting film shield has been proposed to enhance the robustness of the SQUID magnetometer with respect to magnetic noise. And its effectiveness has been confirmed by the experiments. In this paper, we performed the electromagnetic field simulation by using the 3D edge FEM to confirm the effectiveness of the superconducting film shield by the simulation. The SQUID magnetometer and the superconducting film shield are made of HTS. Therefore, to simulate the HTS, the equivalent electric conductivity was employed. Also, the SQUID ring has Josephson junctions. Commonly, the Josephson junction is considered as a resistor in the field simulation. However, in this paper, it is represented by an equivalent circuit to simulate with high accuracy.

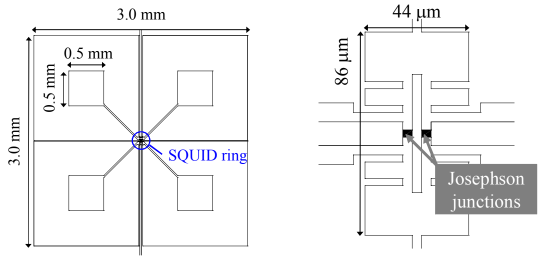
I. INTRODUCTION

A high temperature superconducting quantum interference device (HTS SQUID) is an extremely sensitive magnetic sensor. It can detect a very small amount of magnetic flux and is widely used in various applications. In those applications, the HTS SQUID is often used inside a magnetic shielding room. It is because that the HTS SQUID transitions unstable state in magnetic noise. Therefore, its high robustness with respect to magnetic noise is required to realize stable operation outside the magnetic shielding room. In the paper, we focus on a directly coupled HTS dc-SQUID magnetometer. To enhance the high robustness of the directly coupled HTS dc-SQUID magnetometer, a superconducting thin film shield has been proposed in [1]. The superconducting film shield can effectively protect the SQUID magnetometer from the magnetic noise by the Meissner effect. The effectiveness of the superconducting film shield has been already confirmed by experiments.

In this paper, we have performed an electromagnetic field simulation with the 3D edge finite element method to investigate the effectiveness of the superconducting film shield. The SQUID magnetometer and the superconducting film shield are made of HTS, and the SQUID magnetometer has Josephson junctions. Thus, it is necessary to take into account the characteristics of both the HTS and the Josephson junction. Many ways to simulate the HTS have been proposed [2], however the Josephson junction is often considered as electric resistance in an electromagnetic field simulation. On the other, in an electric circuit simulation [3], the Josephson junction is commonly considered as an equivalent circuit with a resistor, a capacitor and a supercurrent in parallel. Therefore, in the electromagnetic field simulation, the Josephson junction was considered as the equivalent circuit in this paper.

II. SIMULATION MODEL

In this study, the directory coupled HTS dc-SQUID magnetometer with superconducting film shield, which was designed in consideration of the robustness with respect to magnetic noise, was simulated. Fig. 1 shows the geometry of the designed SQUID magnetometer and the SQUID ring. The size of the SQUID magnetometer is 3.0 mm x 3.0 mm, and its thickness is 200 nm. As shown in Fig. 1(b), the Josephson junctions exist in the SQUID ring and have to be taken in account in the simulation. Fig. 2 shows the geometry of the SQUID magnetometer covered by the superconducting film shield with distance of several dozen μm. The size of the superconducting film shield is 1.6 mm x 1.6 mm, and its thickness is also 200 nm. The SQUID magnetometer and the superconducting film shield are made of YBCO thin film.



(a) SQUID magnetometer (b) SQUID ring
Fig.1. Schematic view.

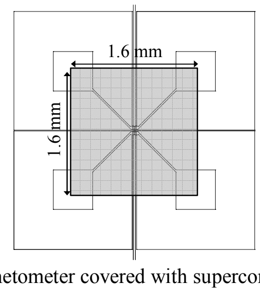


Fig.2. SQUID magnetometer covered with superconducting film shield.

To perform the electromagnetic field simulation, two simulation models were prepared, the Non-shielding model and the Shielding model. Literally, the Non-shielding model has no superconducting film shield, on the other hand the Shielding model has the square superconducting film shield covering the SQUID magnetometer. The distance between the SQUID magnetometer and the superconducting film shield is varied with 10-μm increments in order to investigate the effectiveness of the magnetic shield. Fig. 3 shows an example of the schematic view of the Shielding model. The size of the simulation region, including the surrounding air, is 15 mm x 15 mm x 7.5 mm. As shown in Fig. 3, a bias current flows into the SQUID ring.

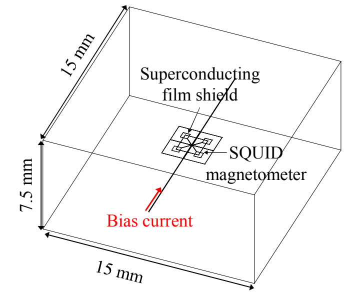


Fig.3. Schematic view of the simulation model.

III. Simulation Method

A. Governing equations

As an electromagnetic field simulation method, the 3D edge finite element method (FEM) is employed. The governing equations derived from Maxwell's equations are

$$\mathrm{rot}(\nu\,\mathrm{rot}\boldsymbol{A}) = \boldsymbol{J}_0 + \boldsymbol{J}_{\mathrm{sc}}\,, \tag{1}$$

$$\mathrm{div}(\boldsymbol{J}_0 + \boldsymbol{J}_{\mathrm{sc}}) = 0\,, \tag{2}$$

where ν is the magnetic reluctivity, $\boldsymbol{A}$ is the magnetic vector potential, $\boldsymbol{J}_0$ is the bias current density of the SQUID magnetometer, and $\boldsymbol{J}_{\mathrm{sc}}$ is the supercurrent density caused by the Meissner effect in the SQUID magnetometer and the superconducting film shield, respectively. The supercurrent density is calculated from the following relation:

$$\boldsymbol{J}_{\mathrm{sc}} = [\boldsymbol{\sigma}_{\mathrm{sc}}]\boldsymbol{E}_{\mathrm{sc}} = \begin{bmatrix} \sigma_{\mathrm{sc},a} & 0 & 0 \\ 0 & \sigma_{\mathrm{sc},b} & 0 \\ 0 & 0 & \sigma_{\mathrm{sc},c} \end{bmatrix} \left(\frac{\partial \boldsymbol{A}}{\partial t} + \mathrm{grad}\phi \right), \tag{3}$$

where $\sigma_{\mathrm{sc},a}$, $\sigma_{\mathrm{sc},b}$ and $\sigma_{\mathrm{sc},c}$ are a, b and c axis components of the equivalent electric conductivity of HTS, respectively. $\boldsymbol{E}_{\mathrm{sc}}$ is the electric field inducing the supercurrent for magnetically shielding, ϕ is the electric scalar potential, and t is the time.

The HTS has a nonlinear characteristic called E-J characteristic. In the simulation, n-value model is employed to express the E-J characteristic [2]. In addition, the c-axis anisotropy of the HTS is also taken into account. Therefore, the equivalent electric conductivity is given by the following equations:

$$\sigma_{\mathrm{sc},a}, \sigma_{\mathrm{sc},b} = \frac{J_{\mathrm{c}}}{|\boldsymbol{E}|}\left(\frac{|\boldsymbol{E}|}{E_{\mathrm{c}}}\right)^{1/n}, \tag{4}$$

$$\sigma_{\mathrm{sc},c} = \mathrm{constant}\,, \tag{5}$$

where $\boldsymbol{E}$ is the electric field, E_{c} is the electric field criterion, J_{c} is the critical current density that is defined by E_{c}, and n is the parameter that is concerned with the nonlinearity. $\sigma_{\mathrm{sc},c}$ is supposed to be large enough to take into c-axis anisotropy.

B. Equivalent of Josephson junction

As mentioned above, the Josephson junctions existing on the SQUID ring has to be taken into account in the simulation. The electric behavior of the Josephson junction changes depending on the condition. In the superconducting state, the resistivity is nearly zero. When applying even a small magnetic field to the Josephson junction, a little resistivity and capacity appear. The electric behavior of the Josephson junction is commonly expressed with the equivalent circuit [3], as shown in Fig. 4. In the simulation, the RSJ model is employed and consists of a supercurrent I_{J}, a resistor R, a capacitor C and a noise current source I_{N}. The circuit equation is given by the following equation:

$$I = I_{\mathrm{J}}\sin\theta + \frac{V}{R} + C\frac{\mathrm{d}V}{\mathrm{d}t} + I_{\mathrm{N}}\,, \tag{6}$$

where I is the total current, V is the voltage and θ is the phase difference across the Josephson junction.

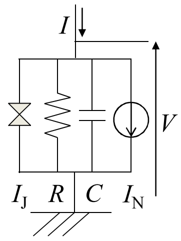


Fig.4. The equivalent circuit of the Josephson junctions, the RSJ model.

IV. Simulation Result

In order to confirm the effectiveness of the superconducting film shield, the amount of the magnetic flux penetrating into the SQUID ring is investigated when applying the external ac magnetic field. Fig. 5 shows the time transition of the amount of the penetrating magnetic flux. It is obvious that the amount of the penetrating magnetic flux can be reduced using the superconducting film shield compared with the Non-shielding model. The simulation result using the RSJ model will have to be compared with one of the resistance model in the full paper.

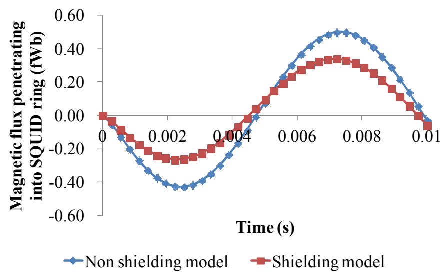


Fig.5. Time transition of the amount of the magnetic flux penetrating into the SQUID ring.

V. References

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